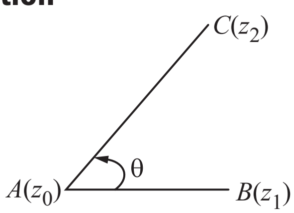
CHAPTER

18 Complex Number-II

Rotation



$$\frac{z_2 - z_0}{|z_2 - z_0|} = \frac{z_1 - z_0}{|z_1 - z_0|} e^{i\theta}$$

Take θ in anticlockwise direction.

Result Related with Triangle

(a) Equilateral triangle:

$\Rightarrow$ $$z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$$

or $$\frac{1}{z_1 - z_2} + \frac{1}{z_2 - z_3} + \frac{1}{z_3 - z_1} = 0$$

(b) Area of triangle ΔABC given by modulus of $\frac{1}{4}\begin{vmatrix} z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \\ z_3 & \bar{z}_3 & 1 \end{vmatrix}$.

Equation of line Through Points z_1 and z_2

$$\begin{vmatrix} z & \bar{z} & 1 \\ z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \end{vmatrix} = 0 \Rightarrow z(\bar{z}_1 - \bar{z}_2) + z_1\bar{z}(z_2 - z_1) + \bar{z}_2 - \bar{z}_1 z_2 = 0$$

$$\Rightarrow z(\bar{z}_1 - \bar{z}_2)i + \bar{z}(z_2 - z_1)i + i(z_1\bar{z}_2 - \bar{z}_1 z_2) = 0$$

Let $(z_2 - z_1)i = a$, then equation of line is $\boxed{\bar{a}z + a\bar{z} + b = 0}$ where $a \in C$ & $b \in R$.

Notes

(i) *Complex slope of line* $\bar{a}z + a\bar{z} + b = 0$ is $-a\frac{1}{\bar{a}}$.

(ii) Two lines with slope μ_1 and μ_2 are parallel or perpendicular if $\mu_1 = \mu_2$ or $\mu_1 + \mu_2 = 0$.

(iii) Length of perpendicular from point $A(\alpha)$ to line $\bar{a}z + a\bar{z} + b = 0$ is $\frac{|\bar{a}\alpha + a\bar{\alpha} + b|}{2|a|}$.

Equation of Circle

(a) Circle whose centre is z_0 and radii $= r$

$$|z - z_0| = r$$

(b) General equation of circle

$$z\bar{z} + a\bar{z} + \bar{a}z + b = 0$$

centre '$-a$' & radii $= \sqrt{|a|^2 - b}$

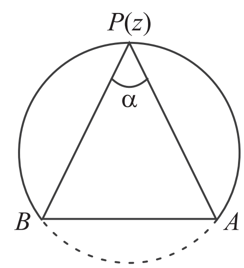
(c) Diameter form $(z - z_1)(\bar{z} - \bar{z}_2) + (z - z_2)(\bar{z} - \bar{z}_1) = 0$

or $$\arg\left(\frac{z - z_1}{z - z_2}\right) = \pm\frac{\pi}{2}$$

(d) Equation $\left|\frac{z - z_1}{z - z_2}\right| = k$ represent a circle if $k \neq 1$ and a straight line if $k = 1$.

(e) Equation $|z - z_1|^2 + |z - z_2|^2 = k$

represent circle if $k \geq \frac{1}{2}|z_1 - z_2|^2$



(f) $\arg\left(\frac{z - z}{z - z}\right) = \alpha \quad 0 < \alpha < \pi, \alpha \neq \frac{\pi}{2}$

represent a segment of circle passing through $A(z_1)$ and $B(z_2)$.

Standard LOCI

(a) $|z - z_1| + |z - z_2| = 2k$ (a constant) represent

(i) If $2k > |z_1 - z_2|$ $\Rightarrow$ An ellipse

(ii) If $2k = |z_1 - z_2|$ $\Rightarrow$ A line segment

(iii) If $2k < |z_1 - z_2|$ $\Rightarrow$ No solution

(b) Equation $||z - z_1| - |z - z_2|| = 2k$ (a constant) represent

(i) If $2k < |z_1 - z_2|$ $\Rightarrow$ A hyperbola

(ii) If $2k = |z_1 - z_2|$ $\Rightarrow$ Union of two ray

(iii) If $2k > |z_1 - z_2|$ $\Rightarrow$ No solution